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The potential possibility of nonlinear recurrence methods application for posttraumatic stress disorder investigation

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ABSTRACT

This article describes methods of nonlinear physics related to recurrent analysis that may be useful in studying the effect of posttraumatic stress disorder on sleep disorders. Traditional pharmacological and psychotherapeutic approaches widely used to treat post-traumatic stress disorder require long-term and painstaking work, combining the joint efforts of clinical specialists and the patient. The versatility and variability of the clinical picture of this disease makes the diagnosis and treatment of post-traumatic stress disorder syndromes particularly difficult. In particular, only in International Classification of Diseases 11th Revision was complex post-traumatic stress disorder isolated from the general group of dissociative disorders. However, one of the few unifying characteristics for such patients is significant disruption of night sleep. Currently, mathematical methods, pumped from nonlinear physics, are often used to analyze physiological signals and assess the condition of patients with various diseases, including depression, chronic migraines, and apnea syndrome. However, recurrent analysis has not been used to date in the study of post-traumatic stress disorder. We are confident, based on the successful application of this method to the study of patients with migraines, orthodontic disorders, and sleep disorders, that this is a major omission and scientists

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working on the problem of post-traumatic stress disorder should pay close attention to the methods proposed in this article for a comprehensive study of the problem. Careful application of the proposed methods will undoubtedly contribute to the study of the effect of various psychiatric diseases on sleep, including posttraumatic stress disorder, and will help to develop more advanced methods of gentle rehabilitation.

Key Words: sleep, polysomnography, nonlinear dynamics, physiological signals, recurrent analysis

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Introduction

Today, diagnoses such as post-traumatic stress disorder (PTSD) and/or complex post-traumatic stress disorder (cPTSD) are more relevant than ever in therapy, neurology and psychiatry, in light of the current global political and social situation. PTSD is a mental illness that significantly affects the quality of life of people suffering from it. It is described as a complex of symptoms caused by anxiety that occurs after a traumatic event. Simple post-traumatic disorder becomes complex when an individual, in addition to other post-traumatic symptoms, experiences self-devaluation [1]. PTSD affects many biological systems, such as brain activity cycles and neurochemical reactions, as well as cellular, immune, endocrine and metabolic functions. In the general human population the overall prevalence of these disorders is 1-15%, and among those wounded in military operations [2], the percentage increases to 20-30% [3], and in psychiatric institutions, the prevalence of PTSD can reach 50% [4]. Patients suffering from PTSD are at increased risk of suicide attempts [5, 6] and are more likely to experience difficulties in social relationships [7]. Currently, mathematical methods, pumped from nonlinear physics, are often used to analyze physiological signals and assess the condition of patients with various diseases, including depression, chronic migraines, and apnea syndrome. However, recurrent analysis has not been used to date in the study of PTSD. We are confident, based on the successful application of this method to the study of patients with migraines, orthodontic disorders, and sleep disorders, that this is a major omission and scientists working on the problem of PTSD should pay close attention to the methods proposed in this article for a comprehensive study of the problem.

Various events, such as hospitalization and medical procedures to which a person is subjected, such as non-invasive ventilation, can predispose to the occurrence of these pathologies [8]. In addition, this disorder is very common among military and civilians affected by counter-terrorism operations, military actions and/or other forms of violence, in the context of which it was identified as a separate diagnosis [3, 9]. Moreover, in the context of the COVID-19 pandemic, this disorder has spread among medical and social workers who have borne the brunt of anti-epidemic measures [10]. Traditional pharmacological and psychotherapeutic approaches widely used to treat PTSD require long-term and painstaking work, combining the joint efforts of clinical specialists and the patient. The versatility and variability of the clinical picture of this disease makes the diagnosis and treatment of PTSD syndromes particularly difficult. In particular, only in International Classification of Diseases 11th Revision (ICD

11) was cPTSD isolated from the general group of dissociative disorders [11]. However, one of the few unifying characteristics for such patients is significant disruption of night sleep [12].

Sleep is one of the points of attraction in interdisciplinary neuroscience and branches of fundamental medicine, from neuro- and psychophysiology to therapy. The quality and duration of sleep directly affect immunity, the preservation of cognitive functions and, in general, the maintenance of normal vital functions of the body [13]. Studies of electrophysiological signals of brain activity, the cardiovascular system and other functional systems during sleep are a powerful direction in the development of neuroscience, where methods of nonlinear dynamics are increasingly used for data processing. For example, today mathematical modeling of the interaction of the respiratory, cardiovascular and central nervous systems phenomenologically demonstrates the development of destructive processes associated with an increase in blood pressure [14] and the occurrence of cognitive impairment in obstructive sleep apnea syndromes [15, 16]. Moreover, studies of the features of oscillatory activity in the microstructure of night sleep make it possible to observe early markers of the development of neurodegenerative diseases [17-19], mental disorders [20, 21] and some somatic disorders [22, 23].

At the same time, the objective map of nocturnal sleep disorders from the point of view of polysomnography (PSG) is still covered with a large number of blank spots. Research aimed at establishing the relationship between sleep and PTSD is at an early stage. An empirically substantiated theory and mathematical model of this relationship have not yet been created, but this relationship is very strong [12, 24]. Today, studies of sleep disorders in PTSD include an analysis of the prevalence of sleep onset disorders, the frequency of nightmares, the content of nightmares, disorders in the paradoxical stage of rapid eye movement (REM) sleep (in particular, the development of motor disorders associated with increased muscle tone), changes in the threshold of arousal during sleep, motor disorders and respiratory failure during sleep [25]. Apparently, the emphasis on the treatment of nocturnal sleep disorders in PTSD and cPTSD is a beneficial strategy for psychotherapeutic care in these patients, or, in other words, treating nocturnal sleep problems alone also leads to an improvement in the general condition in PTSD [26].

Thus, the role of sleep restoration and control in PTSD is difficult to overestimate, and this is an important area for further elucidation of the factors of disease development and treatment of patients with this diagnosis. In addition, testing under PSG control of the proposed algorithm for physiotherapeutic treatment of sleep disorders based on the analysis and control of biophysical characteristics of signals of functional activity of the body will provide new fundamental data on some aspects of sleep development itself, as a unique phenomenon that unites various classes of living systems.

The paper examines the issue of how correct it is to use nonlinear dynamics methods such as recurrent transformations to diagnose changes occurring during sleep in patients with PTSD and cPTSD in comparison with the conventionally normal sleep of an adult. The study of physiological processes of night sleep in health and pathologies using information technologies attracts the attention of researchers both from the standpoint of assessing its general necessity and the possibility of reducing this time, which is unproductive from an economic and social point of view [27, 28]. On the other hand, broad prospects for the treatment and prevention of diseases are potentially opening up for clinical practice in connection with recent studies that have closely linked the sleep of a living system with the normal functioning of the immune system [29]. Moreover, fluctuations in the permeability of the blood-brain barrier (BBB)

that occur during night sleep in both animals and humans, identified in recent years, give hope for significant advances in neurorehabilitation technologies based on high-tech sleep analysis in real time [30, 31].

Characteristics of the relationship between sleep disorders and PTSD

Since the beginning of the 20th century, the number of different types and the total number of nocturnal sleep disorders has been constantly increasing. Such dynamics are caused by the increase in light pollution in cities and, in general, opportunities to “distract” from sleep, and, at the same time, by the growth in the power and number of stress factors in the social organization of modern urban life, which destroy the normal ability to have a full night’s sleep and the normal sleep structure [32]. Despite a significant number of ongoing studies, there is still no unified understanding of such narrow points as, in particular, the relationship between the states of cognitive functions and sleep structure [33, 34], sleep in chronic pain [35], sleep disorders, primary and concomitant with other diseases in patients [36, 37]. In particular, an example of such a lack of complete clarity of the relationship between general pathologies and sleep disorders is PTSD.

Although the full understanding of the pathophysiological mechanisms underlying distress remains incomplete, it is generally recognized that key mediators in stress-related disorders involve the activation of the hypothalamic-pituitary-adrenal (HPA) axis, leading to glucocorticoid release, and the sympathoadrenal (SA) system, responsible for the secretion of adrenaline and noradrenaline. Different stressors impact the HPA and SA systems in varying ways, and the intensity and outcome of these responses are determined by the overall homeostatic state of the organism – shaped by genetic factors, internal and external environmental conditions, and the regulatory programming of glucocorticoids, biogenic amines, and other bioactive substances [38, 39].

Beyond the well-established HPA axis activation in response to stressors, research has identified that proinflammatory cytokines – such as interleukin-1, tumor necrosis factor, and interleukin-6 – can also stimulate the hypothalamus, contributing to the stress response [40]. Interestingly, the development of PTSD, particularly with severe clinical manifestations, is often accompanied by reduced cortisol levels in the acute aftermath of trauma [41–44]. Furthermore, this reduction in circulating glucocorticoids is currently viewed as a potential objective biomarker for the onset of PTSD [45, 46]. In other words, PTSD symptoms apparently correlate with those arising as a result of uncontrolled growth of proinflammatory factors, in particular glucocorticoids, caused by a distressing situation. Glucocorticoid receptors are found in almost all nuclear cells, but the density of glucocorticoid receptors is especially high in the brain, in particular in the hippocampus [47]. Within the framework of PTSD pathogenesis, the emerging neuroinflammation has the character of a pathological uncontrolled chronic process. It is possible that there is positive feedback between the chronicity of such neuroinflammatory processes in different areas of the brain and the occurrence of disturbances in the normal permeability of the BBB [48]. It is important to note that current data regarding the direct impact of stress on the BBB remain inconsistent. For instance, P. Esposito et al. reported that acute immobilization stress in rats led to increased BBB permeability in the diencephalon and cerebellum, while no such changes were observed in the cerebral cortex [49]. Conversely, M. Roszkowski et al., after applying various acute and chronic stress models in mice, did not observe any significant alterations in BBB permeability [50].

In individuals diagnosed with PTSD - as well as in those with schizophrenia and depression - both structural and functional disruptions have been identified in neural pathways linking the hippocampus and prefrontal cortex [51]. Moreover, it has been established that the prefrontal cortex, hippocampus, amygdala, locus coeruleus, and several other brain regions play key roles in the development and persistence of pathological anxiety [52, 53], and are also critically involved in the pathogenesis of depression [54]. At the same time, the locus coeruleus is one of the leading centers of the central nervous system regulating sleep and wakefulness processes. Not least, the sleep disorders observed in PTSD may be associated with neuroinflammatory processes, including in this area of the brain [52].

At the same time, not many works are devoted directly to a full analysis of polysomnographic studies of PTSD patients. This space of the scientific map still shows many blank spots. However, it is already obvious that lack of sleep, disturbances in its structure and microstructure lead to further chronization of problems with consolidation of traumatic memories and an increase in the general level of anxiety of the patient [55]. Interestingly, sleep disruption immediately after, as well as prior to trauma exposure could both increase the risk of PTSD development, suggesting a perpetual circle with pre-existing sleep disturbances increasing the risk for PTSD and vice versa [56-60]. Posttraumatic sleep and circadian disruptions, in turn, affect the neuroendocrine, immune and autonomic systems, leading to impaired adaptive mechanisms, increased sensitivity to stress, and thus may be a cause or at least a powerful factor in the development of stress-related disorders and PTSD in particular [59, 60, 61]. Thus, assessment of sleep quality and circadian patterns should be a priority in the routine clinical assessment of individuals exposed to distress factors and trauma.

Repantis et al. suggest a potentially important role for objective PSG monitoring of sleep stages in individuals in acute distress on the first night after trauma [62]. Identification of objective sleep-related functional parameters in trauma using easily applicable electroencephalography (EEG) devices may improve the ability to correctly predict the potential development of PTSD and guide the way to new sleep interventions to prevent PTSD. In addition, the authors suggest a potential role for modulatory interventions during REM sleep in the prevention of PTSD, such as behavioral sleep deprivation and selective pharmacological (e.g., serotonergic, noradrenergic, cholinergic) suppression or enhancement of REM sleep. Moreover, there is work devoted to the disruption of normal chronorhythms of the body due to acute distress and the occurrence of PTSD, for example, there is evidence that sleep and circadian disruption may represent a vital pre-existing risk factor in the prediction of PTSD development and circadian dysregulation after trauma exposure may represent a core feature of trauma-related disorders mediating enduring neurobiological correlates of traumatic stress through a loss of the temporal order at different organizational levels [60, 63].

At the same time, classical PSG analysis requires specific equipment, premises and an expert - a somnologist, which makes these studies a very expensive and complex procedure. An alternative to classical PSG analysis can be provided by the development of automatic systems based on information technologies using methods of nonlinear physics, artificial intelligence and machine learning, allowing to recognize various stages of sleep and determine pathological changes in the microoscillatory structure of sleep without the participation of a clinical specialist. Good prospects for the development of realistic systems of such analysis are provided by methods of recurrent analysis.

Classical recurrent analysis in problems of polysomnography data processing

Currently, neuroscience uses a large number of nonlinear dynamics methods for processing physiological signals. One of the simplest and most versatile is recurrent analysis, which allows one to establish relationships and correlations between signals in complex distributed systems [64]. Recurrent analysis can be used for both stationary signals and chaotic or noisy signals. In particular, recurrent analysis allows one to identify similar structures in various signals, including EEG, electrocardiography (ECG), and photoplethysmogram (PPG) signals, which form the basis of the PSG recording [65, 66]. It is well suited for processing night sleep data and identifying anomalies in sleep structure, since it is focused on identifying relationships between different signals [67]. Various modifications of the basic analysis and calculation of accompanying metrics make this method very versatile.

The implementation of recurrent analysis is quite simple from a mathematical point of view. The first step is to construct a recurrent matrix, each element of which is determined by the following formula [66]:

$$RP_{i,j} = \theta(\varepsilon - |x_i - x_j|), \quad i, j \in 1, \dots, N \quad (1)$$

Where RP – recurrent rate, ε is neighborhood of the time series value under consideration, determined empirically, x_i and x_j are the elements of the data series with the corresponding times i and j , N is the number of elements of the series, θ is the Heaviside function, which results in 0 if the argument is negative and 1 if it is non-negative [66]:

$$\theta(z) = \begin{cases} 0, & \text{if } z < 0 \\ 1, & \text{if } z \geq 0 \end{cases} \quad (2)$$

Based on this formula, we can obtain a recurrent matrix consisting of zeros

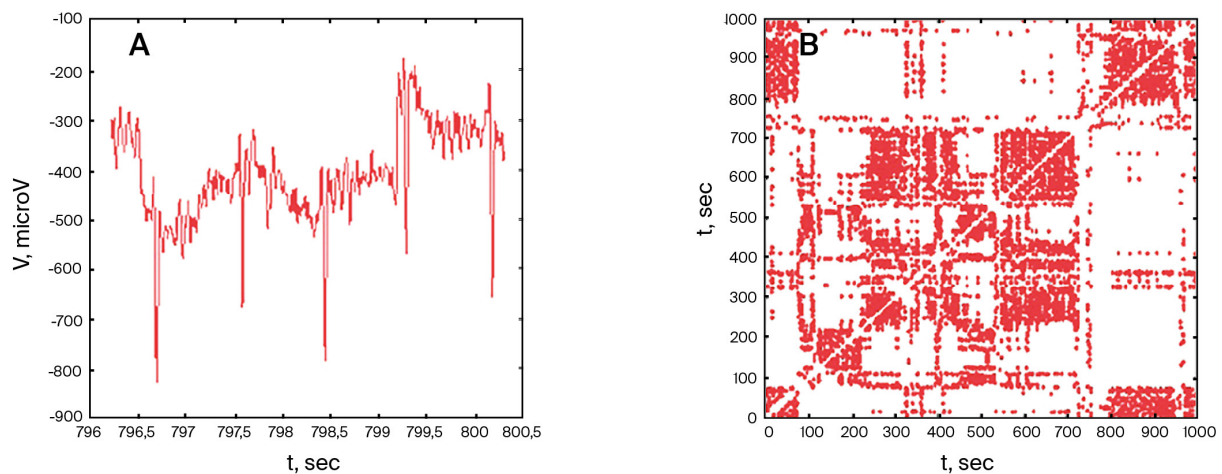
$$\mathbf{R}^X = \begin{matrix} & \begin{matrix} i-1 & i & i+1 & & j-1 & j & j+1 \end{matrix} \\ \begin{matrix} i-1 \\ i \\ i+1 \\ j-1 \\ j \\ j+1 \end{matrix} & \begin{pmatrix} 1 & \vdots & & & \vdots & 0 & \vdots \\ \vdots & 1 & \vdots & \dots & 0 & 1 & 0 \\ & \vdots & 1 & & \vdots & 0 & \vdots \\ \vdots & 0 & \vdots & & 1 & \vdots & \\ 0 & 1 & 0 & \dots & \vdots & 1 & \vdots \\ \vdots & 0 & \vdots & & \vdots & \vdots & 1 \end{pmatrix} \end{matrix}$$

and ones, which has the following form, shown in Figure 1.

FIG. 1. General form of a recurrent matrix

In Figure 1, we can see that each nonzero element with the number i, j or j, i corresponds to the case when the distance between the elements x_i and x_j is less than ε , or, in other words, the element x_j is in the given ε -neighborhood of the element x_i . From the obtained recurrent matrix, we can obtain a recurrent

diagram by coloring all the points with the moments of time that coincide with the numbers of nonzero elements in the matrix and additionally excluding the main diagonal from consideration, since it will always be filled with ones [66]. An example of such a diagram for a short fragment of the EEG recording of one of the experiments with sleep recording is shown in Figure 2. Time is plotted



on both axes of the obtained diagram, which increases to the right and upward.
FIG. 2. Electroencephalography signal recorded during sleep (A) and its corresponding recurrence diagram (B)

The most important parameter when using recurrent analysis is the size of the ε -neighborhood, for this reason its selection is approached with special attention. If the ε -neighborhood is too small, then the number of ones in the recurrent matrix may be very small or may not be there at all, then it is impossible to learn anything about the dynamics of the system under consideration. On the other hand, if the ε -neighborhood is too large, then most of the time implementation points will be included in the neighborhood of each of the points under consideration, thus the recurrent matrix will be filled mainly with ones, which again leads to low information content when studying the system due to a large number of artifacts. It is also necessary to take into account the influence of noise, which can distort the structure of the recurrent diagram. Thus, there is a problem of adequately choosing the size of the ε -neighborhood for the systems under study.

The literature suggests various empirical methods for selecting the ε -neighborhood value, depending on the type of system being studied. The ε -neighborhood can be selected depending on the maximum diameter of the phase space, the density of points in the recurrence diagram, and the signal-to-noise ratio [67, 68]. There are no objective criteria that would allow one to always universally select the ε -neighborhood value for any system being studied, and therefore the choice of the method for determining ε often changes for each individual system. When working with PSG records, the value of the ε parameter was calculated empirically so that the density of points on the recurrence diagram was about 1% (in accordance with N Marwan et al.) for the vast majority of EEG, ECG, or PPG signals being studied [66].

In recurrence diagrams, structures of different types can be observed depending on the system under consideration. In total, eight main patterns can

be observed in recurrence diagrams [69]:

- Uniform filling of the recurrence diagram with points. Such a structure is typical for stationary systems whose relaxation time is small compared to the time covered by the recurrence diagram.
- Periodic structures. They are characteristic of systems with periodicity. In particular, for single-frequency signals, the recurrent diagram will look like a lattice of diagonal lines, the lattice period will correspond to the oscillation period. For multi-frequency signals, the recurrent diagram will be obtained from the superposition of lattices of different periods, corresponding to each of the signal frequencies, due to which a more complex, but still periodically repeating structure can be obtained.
- A gradual decrease in the number of points on a recurrence diagram with distance from the main diagonal, attenuation in the upper left and lower right corners can be observed in systems with variable parameters, that is, in non-stationary ones.
- White areas or stripes are a signal that the system is experiencing sudden changes in the system dynamics that differ greatly from the average time implementation.
- Isolated points can be observed in the presence of significant fluctuations in the system or when considering an uncorrelated random process.
- Diagonal lines parallel to the main diagonal are characteristic of systems whose evolution is the same in different periods.
- Diagonal lines perpendicular to the main diagonal are observed in systems in which evolution is the same, but in reverse time.
- Vertical and horizontal lines are observed when the system does not change over time or changes very slowly, and are also an indicator of a laminar process in the system.

In addition, there are a number of recurrence analysis measures, some of which are based on counting the number of diagonal lines, and some are associated with counting vertical lines.

Measures of recurrence analysis based on diagonal lines

The ratio of recurrence points that form diagonal structures (of at least length l_{\min} – threshold of lines that are formed by the tangential movement) to all recurrence points [66]:

$$DET = \frac{\sum_{l=l_{\min}}^N lP(l)}{\sum_{l=1}^N lP(l)} \quad (3)$$

is introduced as a measure of determinism (DET) (or predictability) of the system. Where $P(l)$ (in general it also depends on ε) – is a diagram of diagonal lines of length l : [66]:

$$P(l) = \sum_{i,j=1}^N (1 - R_{i-1,j-1}(\varepsilon)) (1 - R_{i+1,j+1}(\varepsilon)) \prod_{k=0}^{l-1} R_{i+k,j+k}(\varepsilon) \quad (4)$$

The average line length of diagonal lines (L) which means that the attractor trajectories in the phase space remain close for a long time, relative to the other side, can be determined by the formula [67]

$$L = \frac{\sum_{l=l_{min}}^N lP(l)}{\sum_{l=l_{min}}^N P(l)} \quad (5)$$

The REM measure indicates how quickly the trajectory segments diverge and is related to the exponential divergence of the phase space trajectory [66]:

$$DIV = 1 / \max([l_i]_{i=1}^{N_l}) \quad (6)$$

For demonstrate complexity of the recurrence diagram with respect to the diagonal lines use are quantitative measurements of the entropic characteristics of systems, in particular, those related to Shannon entropy (ENTR), as $p(l) = P(l)/N_p$ that haw follow view

To demonstrate the complexity of the recurrence diagram with respect to the diagonal lines, quantitative measurements of the entropic characteristics of systems are used, in particular, those related to ENTR, as $p(l) = P(l)/N_p$, which follows from the representation [66]:

$$ENTR = -\sum_{l=l_{min}}^N p(l) \ln p(l), \quad (7)$$

Recurrence Analysis Measures Based on Vertical Lines

Laminarity (LAM) is calculated in a similar way to DET [66],

$$LAM = \frac{\sum_{v=v_{min}}^N vP(v)}{\sum_{v=1}^N vP(v)} \quad (8)$$

This measure demonstrates the frequency of occurrence of laminar structures in the system. Here $P(v)$ - total number of vertical lines of length v in a recurrence diagram [66]

$$P(v) = \sum_{i,j=1}^N (1 - R_{i,j})(1 - R_{i,j+v}) \prod_{k=0}^{v-1} R_{i,j+k} \quad (9)$$

In addition, the metrics of trapping time (TT) and length of the longest vertical line (v_{max}) are often used, which are calculated as [66]

$$TT = \frac{\sum_{v=v_{min}}^N vP(v)}{\sum_{v=v_{min}}^N P(v)}, v_{max} = \max([v_i]_{i=1}^{N_v}), \quad (10)$$

TT also called the capture time. This metric estimates the average time the system will stay in a certain state. Unlike measures based on diagonal lines, these measures are able to detect «chaos-to-chaos» transitions. Therefore, they allow one to study intermittency even for rather short and non-stationary data series [66].

Applied Application of Recurrent Analysis Methods in Sleep Research Problems

Thus, we can conclude that recurrent diagrams allow us to quite fully and deeply study the dynamics of systems of the most diverse nature. As practice shows, even such simple measures can help in studying the dynamics of sleep. For example, the Emelyanova et al. article shows that sleep stages are

characterized by different values of the recurrent indicator [70]. The metric of the recurrent indicator is one of the main ones and is the sum of all non-zero elements in the recurrent matrix [66]:

$$RR = \sum_{i=1}^N \sum_{j=1}^N R_{i,j} \quad (11)$$

For REM sleep stages, the recurrent index increases, while for slow sleep stages 3 and 4, the recurrent index decreases significantly. For slow sleep stages 1 and 2, the index remains normal, i.e., on average, it corresponds to wakefulness. These simple patterns not only help to create a mathematically simple algorithm for automatic hypnogram marking, but also to conduct more in-depth studies of various sleep disorders. For example, with a high apnea/hypopnea index, significant changes in the dynamics of recurrent indices during the night are noticeable for different sleep stages [71].

Statistical analysis of changes in recurrent indicators can be a powerful tool for finding sleep disorders caused by various problems, including PTSD. Thus, based on the median and average value, using modern machine learning methods, it is possible to identify groups with normal sleep and with sleep-disordered breathing [72]. Such methods can be used both for early diagnosis of the disorder and for monitoring the rehabilitation process.

An equally important method of processing physiological signals using recurrent analysis is the use of joint recurrent indices and cross-recurrent indices. For signals $x(t)$ and $y(t)$, the values of which are known at the same moments of time t_i , where $i = 1, \dots, n$, the cross-recurrent rate (CRR) can be found using the formula [66]:

$$CRR = \frac{1}{N^2} \sum_{j=1}^N \sum_{i=1}^N \theta(\varepsilon - ||y(t_i) - x(t_j)||), \quad (12)$$

The formula for finding the joint recurrent rate (JRR) is slightly different [66]:

$$JRR = \frac{1}{N^2} \sum_{j=1}^N \sum_{i=1}^N \theta(\varepsilon - ||x(t_i) - x(t_j)||) \theta(\varepsilon - ||y(t_i) - y(t_j)||), \quad (13)$$

The CRR and JRR indicators have fundamentally different meanings. Thus, the value of the CRR increases if at times t_i and t_j the values of the two signals are in the same ε -neighborhood. For the value of the JRR to increase, it is necessary that the pairs of signal values $(x(t_i), x(t_j))$ and $(y(t_i), y(t_j))$ be close (within the ε -neighborhood). In this case, the values of the signals $x(t_i)$ and $y(t_i)$ may differ greatly from each other.

These indices can be used simultaneously to compare the dynamics of physiological signals to determine the degree and objective characteristics of sleep disturbance. It can be expected that the CRR will show the degree of complete synchronization of signals when their values, taking into account the normalizations, coincide. Whereas the JRR will allow us to detect deeper connections between signals when both signals simultaneously change their dynamics.

The calculation of JRR and CRR is especially useful for comparing several channels with each other. However, there is also a modification of the JRR method that estimates the number of repetitions in one channel during identical events. For cognitive tests, this method works similarly to the idea of constructing evoked potentials [73].

In this case, identical types of events are compared with each other and the average JRR is calculated. In the case of PSG processing, sleep stages can be

used as identical events, calculating the average index for each. This will allow us to estimate the number of returns for each channel for each sleep stage. At the same time, a high value of the index, as a rule, indicates the presence of stable patterns in physiological signals. Thus, the use of this method for patients with PTSD will allow us to consider how and in which channels the destruction of habitual sleep patterns occurs first.

Conclusion

This article describes methods of nonlinear physics related to recurrent analysis that may be useful in studying the impact of PTSD on sleep disorders. Currently, mathematical methods, pumped from nonlinear physics, are often used to analyze physiological signals and assess the condition of patients with various diseases, including depression, chronic migraines, and apnea syndrome. However, recurrent analysis has not been used to date in the study of PTSD. We are confident, based on the successful application of this method to the study of patients with migraines, orthodontic disorders, and sleep disorders, that this is a major omission and scientists working on the problem of PTSD should pay close attention to the methods proposed in this article for a comprehensive study of the problem. The proposed methods are very flexible and allow one to evaluate both the overall dynamics of polysomnographic data and to identify sleep stages, consider their changes in case of serious circadian rhythm disturbance, and determine the degree of destruction of normal sleep patterns. Methods based on recurrent analysis are usually not associated with complex mathematics and do not require much time for calculations, unlike frequency methods. The methods are flexible enough to conduct simultaneous analysis of the entire PSG record, including EEG, ECG, and PPG. Careful application of the proposed methods will undoubtedly contribute to the study of the effects of PTSD on sleep and will help to develop more advanced methods of gentle rehabilitation.

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